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like manner we show that a_2, a_3, ...a_n must all be true of quotient, and as the expression is symmetrical with respect to these, the value must be

$$a_1+a_2+a_3+\ldots\ldots+a_n.$$

GEOMETRY.

87. **Proposed by WALTER HUGH DRANE, A. M., Professor of Mathematics, Jefferson Military Academy, Washington, Miss.**

Given any two straight lines in space, AB, CD, which do not intersect. So construct upon one of the lines as base, a triangle, having its vertex in the other line, such that its perimeter shall be a minimum.

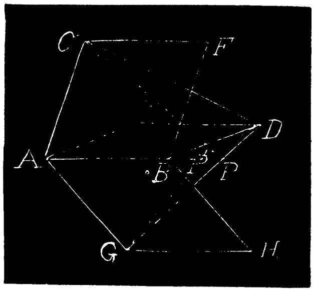
I. Solution by the PROPOSER.

Let AB and CD be the given straight lines. Pass planes through the line AB and the points C and D. In the plane of $ABDE$, inclined the same way and making the same angle with the line AB as $ABFC$, construct a parallelogram equal to $CABF$. Draw DG intersecting AB produced in P. Join PC. Then PCD is the required triangle.

Proof. Take any other point P' in the line AB. Join $P'D$, $P'C$, and $P'G$. Triangle $P'CA$=triangle $P'GA$ and triangle PCA=triangle PGA. Two sides and included angle being equal in each case. $\therefore$ $P'C=PG$ and $PC=PG$.

Now $P'D+P'G>PD+PG$.

$\therefore$ $P'D+P'C>PD+PC$. Q. E. D.



By passing planes through CD and the points A and B, by a similar construction we may construct a minimum-perimeter triangle upon AB as base with its vertex in CD.

Also solved by *F. R. HONEY*.

II. Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.

Let $x+b(a-x)=cy$ be the equation to EF with AB and AY as axes, and AB, EF the given lines, $AB=a$. Then if $\cot\theta+b\cot\varphi=c$ the vertex C will move on EF. Let $AC=r$, $CB=s$.

$$\text{Then } r+s=\frac{a(\sin\theta+\sin\varphi)}{\sin(\theta+\varphi)}=\text{minimum}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1).$$

$$\cot\theta+b\cot\varphi=c\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2).$$

$$\text{From (1), } \frac{d\theta}{d\varphi}=-\frac{\sin\theta}{\sin\varphi}, \text{ from (2), } \frac{d\theta}{d\varphi}=-\frac{b\sin^2\theta}{\sin^2\varphi}.$$

$\therefore$ $\sin\varphi=b\sin\theta$, this in (2) gives

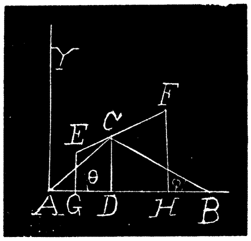
$$\cot\theta=\frac{b^2+c^2-1}{2c},\ \cot\varphi=\frac{c^2-b^2+1}{2bc},\ \cot\theta+\cot\varphi=\frac{AD+DB}{DC}=a/DC,$$

$$\therefore\ DC=\frac{2abc}{b^3-b^2-b+bc^2+c^2+1}.$$

DC must be $> EG$ and $<HF$.

Let $AG=m$, $EG=n$, $AH=h$, $HF=k$.

Then $b=\dfrac{hn-km}{ak+hn-an-km}$, $c=\dfrac{a(h-m)}{ak+hn-an-km}$, b must be less than unity or EF may intersect AB.



Let $m=n=1$, $h=a=10$, $k=4$. $\therefore b=\frac{1}{6}$, $c=\frac{5}{2}$, $DC=\frac{36}{35}$.

$\cot\varphi=\frac{131}{15}$, $\cot\theta=\frac{19}{18}$. $\therefore \theta=43°\ 27'\ 6''$, $\varphi=6°\ 31'\ 56''$.

$AD=DC\cot\theta=\frac{38}{35}$.

89. Proposed by **B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.**

Describe a circle tangent to three given circles. [From *Chauvenet's Geometry*, page 318, ex. 213.

Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.; and F. R. HONEY, Ph. B., Instructor in Mathematics in Trinity College, New Haven, Conn.

In figure 2 let L, M, N be the circles radii a, b, c.

With M as center and radius $b-a$ describe a circle, also with N as center and radius $c-a$ describe a circle. Draw a circle through L tangent to the circles last described at T, S then the center of this circle is the center of one of the tangent circles. Similarly we can find seven other tangent circles.

Of the eight circles one is tangent to the three circles externally, one is tangent internally, three are tangent to two externally and one internally, and three are tangent to two internally and one externally.

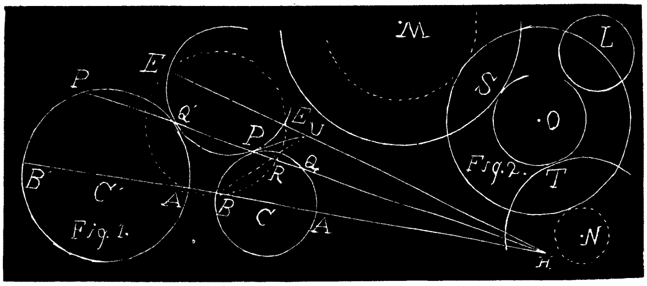


Fig. 1. Fig. 2.

In figure 1, to find a circle passing through a point E and tangent to two circles C, C'. Let H be the point where the external common tangent meets $C'C$ produced. Through $A'BE$ describe a circle cutting EH again in E'. Draw BR meeting HE in U, and draw UP tangent to C, then the circle through PEE' is the circle required.